

Determination of the mass of the $\Lambda\Lambda$ dibaryon by the method of QCD sum rules

S. A. Larin, V. A. Matveev, A. A. Ovchinnikov, and A. A. Pivovarov

Institute of Nuclear Research, USSR Academy of Sciences

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Abstract

The method of QCD sum rules is used to calculate the mass of a hypothetical stable dihyperon H with the quantum numbers of two Λ hyperons, whose existence was predicted by R. L. Jaffe [Phys. Rev. Lett. **31**, 195, 617(E) (1977)] in the framework of the MIT quark-bag model. Within the accuracy of the method of QCD sum rules, which in the present case is $\sim 20\%$, the results obtained here agree with those of Jaffe. However, the method of sum rules does not make it possible to determine whether the mass of the dihyperon H lies above or below the $\Lambda\Lambda$ threshold (i.e., whether H is stable).

The theoretical investigation of multiquark states ($Q^n\bar{Q}^m$, $n+m > 3$) and the experimental search for them may provide important information about the properties of the interaction of quarks at large distances. In particular, major efforts, both theoretical [1, 2] and experimental [3] have been directed at the study of the dibaryon spectrum ($n = 6$, $m = 0$).

In this connection, interest attaches to Jaffe's prediction [1] that there exists a stable six-quark s -wave state – a dihyperon H , which is a singlet with respect to both color and flavor (with strangeness -2) and has $J^P = 0^+$ and mass 2150 MeV. The quantum numbers of H are identical to the quantum numbers of $\Lambda\Lambda$ [two $\Lambda(1115)$ hyperons], and its mass is smaller than the sum of the masses of two Λ hyperons. Therefore H can decay only through the weak channel (consequently, it is stable with respect to the strong interactions).

The prediction that a dihyperon exists was obtained in the framework of the MIT quark-bag model [4]. It is important to test this model prediction solely on the basis of the fundamental principles of QCD. Such a test can be made by means of the method of QCD sum rules, using either the technique of finite-energy sum rules [5] or that of Borel sum rules [6]. The method of QCD sum rules is based on the fundamental principles of QCD, and it has proved its effectiveness in calculations of the masses of mesons [6, 7, 8, 9] and baryons [9, 10, 11].

In the present paper, assuming the existence of the above-mentioned dihyperon, we calculate its mass (with the aim of testing the prediction of Ref. [3]) in the framework of the Borel version of the method of QCD sum rules [6].

According to the method of QCD sum rules, to calculate the H mass we must consider the correlation function

$$\Pi(q) = i \int d^4x e^{iqx} \langle 0 | T\{h(x)h^\dagger(0)\} | 0 \rangle. \quad (1)$$

Here $h(x)$ is a scalar local current with the quantum numbers of the dihyperon H , constructed from

six quark fields, such that its projection onto the real dihyperon state $|H(p)\rangle$ (we are working with the assumption that this state exists) is nonzero:

$$\langle 0|h(0)|H(p)\rangle = \lambda_H, \quad p^2 = m_H^2. \quad (2)$$

Since such a current $h(x)$ is not unique, the question of its optimal choice immediately arises. We recall that the problem of choosing the current already arose [10, 11] in the case of baryons, when the currents are constructed from three quark fields. In our case, when the current is constructed instead from six quark fields, this problem is much more complicated, since the number of independent currents with the given quantum numbers is much greater. The treatment of the current $h(x)$ in the most general form, i.e., in the form of a linear combination of all the independent local operators with the quantum numbers of H , is a very cumbersome problem. Therefore we confine ourselves to the choice of a few of the simplest currents with the required quantum numbers and analyze the dependence of our results on the properties of the studied currents. As in the case of mesons and baryons, we shall construct the current $h(x)$ from quark fields without derivatives.

We shall consider only the following six simplest (from the point of view of the calculation of the correlation function Π) currents with the quantum numbers of the dihyperon H (we recall that H is a singlet with respect to both color and flavor):

$$\begin{aligned} h_1(x) &= \varepsilon^{A_1 A_2 A_3} \varepsilon^{A_4 A_5 A_6} \varepsilon_{a_1 a_2 a_3} \varepsilon_{a_4 a_5 a_6} (\psi_1 C \psi_2)(\psi_3 C \gamma_5 \psi_4)(\psi_5 C \psi_6), \\ h_2(x) &= \varepsilon^{A_1 A_2 A_3} \varepsilon^{A_4 A_5 A_6} \varepsilon_{a_1 a_2 a_3} \varepsilon_{a_4 a_5 a_6} (\psi_1 C \gamma_5 \psi_2)(\psi_3 C \gamma_5 \psi_4)(\psi_5 C \gamma_5 \psi_6), \\ h_3(x) &= \varepsilon^{A_1 A_2 A_3} \varepsilon^{A_4 A_5 A_6} \varepsilon_{a_1 a_2 a_3} \varepsilon_{a_4 a_5 a_6} (\psi_1 C \gamma_\mu \gamma_5 \psi_2)(\psi_3 C \gamma_5 \psi_4)(\psi_5 C \gamma_\mu \gamma_5 \psi_6), \\ h_4(x) &= \varepsilon^{A_1 A_2 A_4} \varepsilon^{A_3 A_5 A_6} \varepsilon_{a_1 a_2 a_3} \varepsilon_{a_4 a_5 a_6} (\psi_1 C \gamma_5 \psi_2)(\psi_3 C \gamma_5 \psi_4)(\psi_5 C \gamma_5 \psi_6), \\ h_5(x) &= \varepsilon^{A_1 A_2 A_4} \varepsilon^{A_3 A_5 A_6} \varepsilon_{a_1 a_2 a_3} \varepsilon_{a_4 a_5 a_6} (\psi_1 C \gamma_5 \psi_2)(\psi_3 C \gamma_5 \psi_4)(\psi_5 C \gamma_5 \psi_6), \\ h_6(x) &= h_5(x) - (1/3)h_2(x), \end{aligned} \quad (3)$$

where $\psi_n = \psi_{A_n}^{a_n}(x)$ is a quark field, a_n is the color index, A_n is the flavor index, the spinor indices are omitted, and a summation over repeated indices is understood; $\varepsilon^{A_1 A_2 A_3}$ and $\varepsilon_{a_1 a_2 a_3}$ are antisymmetric tensors which are invariant with respect to the flavor group $SU^f(3)$ and the color group $SU_c(3)$, respectively; C are the matrices of charge conjugation. [The choice of the current $h_6(x)$ will be discussed below.]

In accordance with the method of QCD sum rules, we shall calculate the correlation functions

$$\Pi_j(q) = i \int d^4x e^{iqx} \langle 0 | T\{h_j(x)h_j^+(0)\} | 0 \rangle \quad (4)$$

by means of Wilson's operator expansion, assuming that the vacuum expectation values of the local operators (the so-called condensates) are nonzero. The calculations must be performed in the region $-q^2 \geq 1 \text{ GeV}^2$. In this region, on the one hand, the effective strong interaction constant α_s is already small enough to calculate the coefficient functions of the operator expansion by perturbation theory; on the other hand, the nonperturbative corrections due to the nonzero vacuum expectation values of the local operators are already quite important.

Because of the great complexity of the calculations of Π_j , we shall confine ourselves to a calculation in the zeroth order in α_s . The fact that this approximation may be sufficient for calculations of the physical quantities in the framework of the method used here is confirmed by the practice of using the method, in particular by the calculation of baryon masses. We calculate Π_j in the linear approximation

in the strange quark mass m_s and in the zeroth order in the masses of the u and d quarks. The results of our calculations are as follows:

$$\begin{aligned}
\Pi_1 &= \Pi_2 = \frac{1}{16} \Pi_3 = \text{const} \cdot \Pi_4 = & (5) \\
&= \left\{ \frac{2}{35} \frac{(-q^2)^7}{7!} \ln \left(\frac{-q^2}{\mu^2} \right) + \frac{2}{15} \frac{(-q^2)^4}{4!} \ln \left(\frac{-q^2}{\mu^2} \right) a^2 - \frac{1}{18} (-q^2) \ln \left(\frac{-q^2}{\mu^2} \right) a^4 \right. \\
&\quad \left. + m_s \left[-\frac{4}{9} \frac{(-q^2)^5}{5!} \ln \left(\frac{-q^2}{\mu^2} \right) a + \frac{4}{27} \frac{(-q^2)^2}{2!} \ln \left(\frac{-q^2}{\mu^2} \right) a^3 + \frac{7}{324} \frac{1}{(-q^2)} a^5 \right] \right\} \frac{64}{(16\pi^2)^5}, \\
\Pi_5 &= \left\{ \frac{23}{35} \frac{(-q^2)^7}{7!} \ln \left(\frac{-q^2}{\mu^2} \right) - \frac{121}{15} \frac{(-q^2)^4}{4!} \ln \left(\frac{-q^2}{\mu^2} \right) a^2 + \frac{121}{36} (-q^2) \ln \left(\frac{-q^2}{\mu^2} \right) a^4 \right. \\
&\quad \left. + m_s \left[\frac{346}{45} \frac{(-q^2)^5}{5!} \ln \left(\frac{-q^2}{\mu^2} \right) a - \frac{242}{27} \frac{(-q^2)^2}{2!} \ln \left(\frac{-q^2}{\mu^2} \right) a^3 + \frac{17}{648} \frac{a^5}{(-q^2)} \right] \right\} \frac{64}{(16\pi^2)^5}, \\
\Pi_6 &= \left\{ \frac{43}{63} \frac{(-q^2)^7}{7!} \ln \left(\frac{-q^2}{\mu^2} \right) - \frac{1081}{135} \frac{(-q^2)^4}{4!} \ln \left(\frac{-q^2}{\mu^2} \right) a^2 + \frac{1081}{324} (-q^2) \ln \left(\frac{-q^2}{\mu^2} \right) a^4 \right. \\
&\quad \left. + m_s \left[\frac{3034}{405} \frac{(-q^2)^5}{5!} \ln \left(\frac{-q^2}{\mu^2} \right) a - \frac{2162}{243} \frac{(-q^2)^2}{2!} \ln \left(\frac{-q^2}{\mu^2} \right) a^3 + \frac{209}{5832} \frac{a^5}{(-q^2)} \right] \right\} \frac{64}{(16\pi^2)^5}.
\end{aligned}$$

Here $a = |16\pi^2 \langle \bar{q}q \rangle|$, isotopic invariance gives $\langle \bar{u}u \rangle = \langle \bar{d}d \rangle = \langle \bar{q}q \rangle$, and we also assume $\langle \bar{s}s \rangle = \langle \bar{q}q \rangle$ for the strange quark condensate, since our analysis has shown that allowance for the difference between $\langle \bar{s}s \rangle$ and $\langle \bar{q}q \rangle$ practically does not alter our final results. In the expressions (5) we have retained only the terms which survive after application of a Borel transformation (see below); in the calculation of the vacuum expectation values of multiquark operators, we have made use of the hypothesis of factorization [4]; the factor $1/(16\pi^2)^5$ corresponds to the fact that the perturbative contribution is represented by five-loop diagrams.

For each correlation function Π_j we can write the dispersion relation (below, we shall drop the index j , since the arguments are valid for all j)

$$\Pi(Q^2) = \int_0^\infty \frac{\rho(s)ds}{s+Q^2} - \text{subtractions} , \quad Q^2 = -q^2 , \quad (6)$$

where the spectral density $\rho(s) = (1/\pi)\text{Im}\Pi(-s-i0)$ behaves as s^7 for large s , so that eight subtractions must be made in the relation (6). Applying a Borel transformation (i.e., the differential operator

$$\hat{B} = \lim_{Q^2, n \rightarrow \infty} \frac{Q^{2n}}{(n-1)!} \left(-\frac{d}{dQ^2} \right)^n |_{Q^2/n=y}$$

which makes the subtractions vanish) to both sides of Eq. (6), we obtain a Borel sum rule of the well known form

$$\Pi^B(y) = \frac{1}{y} \int_0^\infty e^{-s/y} \rho(s)ds , \quad (7)$$

where $\Pi^B(y)$ can be obtained in our approximation from (5) by the action of the operator \hat{B} :

$$\hat{B}Q^{2k} \ln \left(\frac{Q^2}{\mu^2} \right) = (-1)^{k+1} k! y^k, \quad \hat{B} \frac{1}{Q^{2k}} = \frac{1}{(k-1)!} \frac{1}{y^k}. \quad (8)$$

In order to calculate the mass of the dihyperon, it is necessary to choose some phenomenological ansatz for the spectral density $\rho(s)$. The simplest procedure which has proved to be successful in the calculation of meson and baryon masses is to represent ρ as the sum of the pole contribution of the unknown lowest state of the given channel and an effective continuum beginning at some effective threshold W and approximating the contributions of all the higher states. We shall also take a spectral density in the "pole plus continuum" form

$$\rho(s) = \lambda_H^2 \delta(s - m_H^2) + \theta(s - W^2) \rho^{cont}(s), \quad \rho^{cont}(s) = (1/\pi) \text{Im} \Pi(-s - i0) \quad (9)$$

where the spectral density of the continuum is determined as the imaginary part of the complete approximation calculated for Π . Substituting the spectral density (9) into Eq. (7) and transferring the continuum contribution to the left-hand side, we obtain a sum rule for the correlation function Π_1 ,

$$\begin{aligned} & \frac{2}{35} y^7 - \frac{2}{15} a^2 y^4 - \frac{1}{18} a^4 y + m_s \left(-\frac{4}{9} a y^5 - \frac{4}{27} a^3 y^2 + \frac{7}{324} a^5 \frac{1}{y} \right) \\ & - \frac{1}{y} \int_{W^2}^{\infty} e^{-s/y} \tilde{\rho}^{cont}(s) ds = \frac{1}{y} \tilde{\lambda}^2 e^{-m_H^2/y} \end{aligned} \quad (10)$$

and a sum rule for the correlation function Π_6 ,

$$\begin{aligned} & \frac{43}{63} y^7 + \frac{1081}{135} a^2 y^4 + \frac{1081}{324} a^4 y + m_s \left(\frac{3034}{405} a y^5 + \frac{2162}{243} a^3 y^2 + \frac{209}{5832} a^5 \frac{1}{y} \right) \\ & - \frac{1}{y} \int_{W^2}^{\infty} e^{-s/y} \tilde{\rho}^{cont}(s) ds = \frac{1}{y} \tilde{\lambda}^2 e^{-m_H^2/y} \end{aligned} \quad (11)$$

where

$$\tilde{\rho}^{cont}(s) = \frac{(16\pi^2)^5}{64} \rho^{cont}(s), \quad \tilde{\lambda}^2 = \frac{(16\pi^2)^5}{64} \lambda^2.$$

The sum rules for $\Pi_{2,3,4}$ are identical to (10) apart from an unessential overall factor. We have also not written down the sum rule for the correlation function Π_5 , since the coefficients in Π_5 are numerically close to the coefficients in the correlation function Π_6 [see Eq. (5)], so that the results obtained from the sum rule (11) for Π_6 are practically identical to the results of Π_5 .

The sum rules must be considered in an admissible interval of values of y , in which the contributions of the higher power corrections and, at the same time, the contribution of the effective continuum must not be too large. We shall call this interval Ω and impose the following restrictions for its determination: 1) the continuum contribution is less than 50% of the calculated approximation for $\Pi^B(y)$ which means that the continuum contribution is smaller than the contribution of the dihyperon H to the sum rule; 2) the sum of all the nonperturbative corrections, excluding the leading nonperturbative term $\sim \langle \bar{q}q \rangle^2 y^4$, is less than 50% of the calculated approximation for $\Pi^B(y)$. (Although with respect to the dimensions of y the term $\sim m_s \langle \bar{q}q \rangle^2 y^5$ is the leading nonperturbative term, it is natural to include it in the constrained sum, since it is $\sim m_s$ and numerically the case $m_s = 0$ should not differ fundamentally from the case of broken $SU^f(3)$ symmetry, $m_s \neq 0$; moreover, if the term $\sim m_s \langle \bar{q}q \rangle^2 y^5$ is not included in the constrained sum, this has little influence on the results.)

The choice of some *a priori* limit (in our case, 50%) is highly conditional. In the final analysis, its correctness is substantiated *a posteriori* by the consistency of the sum rule in the interval Ω which is found.

Before particularizing the procedure of fitting the sum rules, we shall discuss the choice of the numerical value of the vacuum expectation value $\langle\bar{q}q\rangle$, which depends on the normalization point μ . We shall choose $\mu^2 \sim y \in \Omega$. Then the renormalization-group factor $[\alpha_s(y)/\alpha_s(\mu^2)]^{\delta_n}$ in the coefficient function of a given operator O_n (where δ_n is determined by the anomalous dimensions of O_n and of the current h) becomes ~ 1 for $y \in \Omega$. Therefore we shall neglect the effects of the anomalous dimensions. In our case, the analysis of the interval Ω shows that it is necessary to take $\mu \sim 1$ GeV. If we begin with the value from Ref. 6,

$$|\langle\bar{q}q\rangle|_{\mu \sim 200 \text{ MeV}} = (240 \text{ MeV})^3 = 0.014 \text{ GeV}^3$$

then the application of the renormalization group gives the first estimate $|\langle\bar{q}q\rangle|_{\mu \sim 1 \text{ GeV}} = 0.024 \text{ GeV}^3$. On the other hand, if we recalculate the value of $\langle\bar{q}q\rangle$ according to the PCAC formula $\langle\bar{q}q\rangle = -(1/2)f_\pi^2 m_\pi^2/(m_u + m_d)$, then by using the value $m_u + m_d|_{\mu \sim 1 \text{ GeV}} = 16 \text{ MeV}$ which is now adopted for the masses of the light quarks [12] we obtain the second estimate $|\langle\bar{q}q\rangle|_{\mu \sim 1 \text{ GeV}} = 0.011 \text{ GeV}^3$. It can be seen that those two values differ by a factor ~ 2 .

Our final results depend little on the variation of $\langle\bar{q}q\rangle$ in the indicated interval. This is perfectly natural. The value of $\langle\bar{q}q\rangle$ is the only number which is built into our sum rule (inclusion of the terms $\sim m_s$ should not substantially alter the results). Therefore it follows from dimensional arguments that the mass of the dihyperon is $m_H \sim (|\langle\bar{q}q\rangle|)^{1/3}$. Consequently, m_H should not change by more than $\sim 30\%$ if $\langle\bar{q}q\rangle$ varies by a factor 2.

We must determine three parameters (m_H, W, λ) by fitting the sum rule in expression (10) [or (11)], i.e. on the basis of agreement between the left and right hand sides [which will be denoted by $L(W, y)$ and $R(m_H, y)$, respectively] of the sum rule (10) [or (11)] in the admissible interval Ω . We rewrite the sum rule in the form

$$L(W, y)ye^{m_H^2/y} = \tilde{\lambda}^2. \quad (12)$$

Let $F(m_H, W, y)$ denote the left hand side of Eq. (12). Since $\tilde{\lambda}^2$ must not depend on y , we shall determine the values of m_H and W by requiring the minimal dependence of F on y in the interval Ω . Thus, only two parameters m_H and W are effectively fitted, and $\tilde{\lambda}^2$ is determined simply as the mean value $\bar{F}(m_H, W)$ of the function F in Ω for the values of m_H and W which are found. We introduce a criterion for the determination of m_H and W . We consider the relative deviation of $F(m_H, W, y)$ from its mean value:

$$\Delta = \left| \frac{F(m_H, W, y) - \bar{F}(m_H, W)}{\bar{F}(m_H, W)} \right| = \left| \frac{FL(W, y) - R(m_H, y)}{\bar{R}(m_H, y)} \right|. \quad (13)$$

We shall fix the values of m_H and W by requiring the largest possible interval Ω (the upper limit of Ω depends on W) subject to the condition that the relative error Δ does not exceed a few percent (i.e., the function F is practically a constant in Ω , since it is a smooth function of y). For definiteness, we require that $\Delta < 5\%$.

We analyzed the accuracy of the determination of the parameters on the basis of the fitting procedure described above as follows. The criterion for Δ was varied by a factor 2 in each direction from the given value $\Delta = 5\%$, and the value of the quark condensate $\langle\bar{q}q\rangle$ was varied between the limits described above. The corresponding changes in the values of m_H and W determined from the fit were not more than 20%. Therefore we can say that our sum rule fixes m_H and W with 20% accuracy. The parameter λ^2 is determined with much lower accuracy with an accuracy up to a factor 2. The

greater accuracy in the determination of m_H and W than in that of Δ is natural, since m_H and W appear in the arguments of exponential functions and, consequently, the sum rule is more sensitive to their variation.

Our analysis has shown that the sum rule (10) for the currents $h_1 - h_4$ is poorly fitted (in other words, it is not possible to satisfy the criterion introduced above). At the same time, the sum rule (11) for the current h_6 is well fitted over a rather large interval Ω . In particular, for $\langle \bar{q}q \rangle = -(240 \text{ MeV})^3$ the results of fitting the sum rule (11) are as follows. For $m_s = 0$ we obtain

$$m_H = 2.0 \text{ GeV}, \quad W = 2.8 \text{ GeV}, \quad \tilde{\lambda}^2 = 70 \text{ GeV}^8, \quad (14)$$

and for $m_s = 0.2 \text{ GeV}$ (Ref. 12)

$$m_H = 2.4 \text{ GeV}, \quad W = 3.2 \text{ GeV}, \quad \tilde{\lambda}^2 = 150 \text{ GeV}^8. \quad (15)$$

The y dependences of the left and right hand sides of the sum rule (11) for the parameter values (14) and (15) which we have found are shown in Figs. 1a and 1b, respectively.

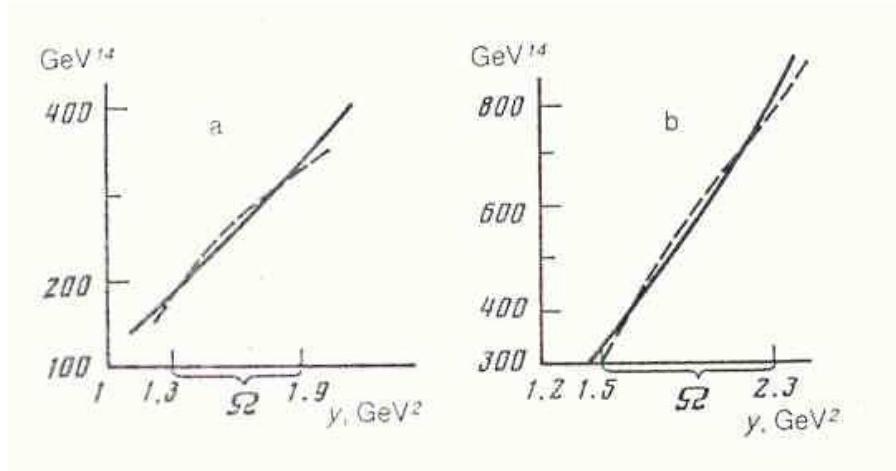


Figure 1: Dependence of the left hand side (continuous curves) and right hand side (broken curves) of the sum rule (11) on y for the parameter values in (14) and (15): a) $m_s = 0$; b) $m_s = 0.2 \text{ GeV}$.

We recall the values obtained by Jaffe [1] for the mass of the dihyperon H (the values of the dibaryon masses obtained in the quark-bag model correspond to poles of the P -matrix [13] and the physical states have masses 100-200 MeV lower than those predicted by this model):

$$m_H|_{m_s=0} = 1.76 \text{ GeV}, \quad m_H|_{m_s \neq 0} = 2.150 \text{ GeV}. \quad (16)$$

It can be seen that the values (14,15) obtained for the dihyperon mass by the method of QCD sum rules agree within the accuracy of this method (in our case, the mass is determined with an accuracy ($\sim 20\%$) with the results (16) of the MIT quark-bag model).

However, there immediately arises the question of what makes the current h_6 , for which the results (14,15) are obtained (the results for h_5 are practically identical to them), preferred over the currents $h_1 - h_4$ [see Eq. (3)] which lead to the "poor" sum rule (10).

In our opinion, the answer is that the current h_6 satisfies the following two "physical" requirements. First, there exists a nonzero nonrelativistic limit for it [i.e. if the quark field $\psi(x)$ is represented in

the standard manner in terms of the small and large components, the term containing only the large component will be nonzero]. On the other hand, the currents h_1 and h_4 do not possess a nonrelativistic limit. We note that in Refs. [10] and [14] it was already pointed out that the existence of a nonrelativistic limit is a desirable property for the construction of currents in working with the method of QCD sum rules.

Second, we represent the current h_6 in the form of a product “singlet \otimes singlet” with respect to color, i.e., in the form $h_6 = \Psi_1(x)\Gamma\Psi_2(x)$, where each of the operators Ψ_1 , and Ψ_2 is a color singlet and Γ is some combination of Dirac γ matrices. Then we obtain the representation (the currents h_j are as a whole singlets with respect to both color and flavor)

$$h_6(x) = (\Psi_{A_3}^{A_4}(x))^\alpha (C\gamma_5)_\alpha^\beta (\Psi_{A_4}^{A_3}(x))_\beta \quad (17)$$

where the color-singlet operator Ψ is a flavor octet:

$$(\Psi_{A_3}^{A_4})_\alpha = \varepsilon_{a_1 a_2 a_3} (\psi_{A_1}^{a_1} C\gamma_5 \psi_{A_2}^{a_2})(\psi_{B_1}^{a_3})_\alpha \varepsilon^{A_1 A_2 B_2} (\delta_{A_3}^{B_1} \delta_{B_2}^{A_4} - (1/3) \delta_{B_2}^{B_1} \delta_{A_3}^{A_4}). \quad (18)$$

Here α and β are spinor indices, and the remaining notation is the same as in (3).

We note that the flavor octet (18) has the quantum numbers of the baryon octet and has been used to calculate the characteristics of baryons by the method of QCD sum rules [10, 11]. Thus, if the current h_6 is represented in the form “singlet \otimes singlet” with respect to color, it has the structure “octet \otimes octet” with respect to flavor (each color singlet is a flavor octet). Since the hyperon Λ is a member of the baryon flavor octet and the quantum numbers of the dihyperon H are identical to the quantum numbers of $\Lambda\Lambda$, it seems natural to construct a flavor singlet $h(x)$ in the form “octet \otimes octet”.

Physically, we can imagine the following picture. If we split the colorless dihyperon H into two colorless clusters and separate them to a large distance, we obtain just two hyperons. However, if the currents $h_1 - h_3$, are represented in the form “singlet \otimes singlet” with respect to color, they will also have the flavor structure “singlet \otimes singlet” (each color singlet is also a flavor singlet), and therefore their structure seems less natural than the structure of h_6 .

As a result, we conclude that the current h_6 is more “physical” than the currents $h_1 - h_4$, since it has a nonrelativistic limit and is constructed as “octet \otimes octet” with respect to flavor (in the sense indicated above). However, the currents $h_1 - h_4$ are “poor” because they either do not possess a nonrelativistic limit (the currents h_1 , and h_4) or have the flavor structure “singlet \otimes singlet” (the currents h_1 , h_2 , and h_3). The current h_5 (we recall that the results for it are practically identical to the results for h_6) has the flavor structure “nonet \otimes nonet”, and the “octet \otimes octet” term in it dominates over the “singlet \otimes singlet” term.

We must make the reservation that formally the method of QCD sum rules dictates *a priori* the only condition for the choice of the current: it must possess the required quantum numbers. However, *a posteriori* (after the fit) positive results were obtained only for the current satisfying the two requirements formulated above. In our opinion, this indicates the existence of a criterion which makes it possible to select the optimal (“physical”) current from the set of currents with the quantum numbers of the given channel. We note that this conclusion is confirmed also by the results of Ref. [15], in which the deuteron mass was calculated by the same method. In that paper, the current was also constructed as “baryon \otimes baryon”, and positive results were obtained only when in the current the term having a nonrelativistic limit dominated over the term not having this limit.

Unfortunately, the relatively low accuracy of our method in the determination of the mass ($\sim 15\%$) does not make it possible to draw a conclusion on the basis of our values (14,15) about whether the mass of the dihyperon H lies below or above the $\Lambda\Lambda$ threshold (equal to 2.23 GeV). (We recall that the high accuracy of the MIT quark-bag model permitted Jaffe to conclude that H lies below the $\Lambda\Lambda$

threshold and is therefore stable.) On the basis of our analysis, we can say that the method of QCD sum rules agrees with the possible existence of a dihyperon H either below or above the $\Lambda\Lambda$ threshold (since in both cases it is possible to represent the spectral density ρ in the form of the sum of a pole contribution from H and a contribution from an effective continuum).

We shall consider in detail the case in which H lies below the $\Lambda\Lambda$ threshold. In this case, if the binding energy $\varepsilon = 2m_\Lambda - m_H$ is sufficiently small (what "small" means will be particularized below), there can occur the interesting effect of resonance enhancement of the contribution of the $\Lambda\Lambda$ state of the continuum [15]. To describe this effect, let us consider the contribution of the state $|\Lambda\Lambda\rangle$ (two free hyperons) to the spectral density:

$$\rho_{\Lambda\Lambda}(q^2) = (2\pi)^3 \langle 0 | h(0) | \Lambda\Lambda \rangle \langle \Lambda\Lambda | h^+(0) | 0 \rangle \delta(q - p_1 - p_2) \quad (19)$$

where $p_1^2 = p_2^2 = m_\Lambda^2$, and an integration over the phase space of the $\Lambda\Lambda$ state is understood. (We note the inclusion of the contributions of the $\Sigma\Sigma$ and ΞN states in the approximation of exact $SU^f(3)$ symmetry with allowance for only the flavor-singlet states does not alter the results.) For the matrix element $\langle 0 | h(0) | \Lambda\Lambda \rangle = f_h(s)$, $s = q^2$, we can write the dispersion relation

$$f_h(s) = \frac{\lambda_H g_{H\Lambda}}{m_H^2 - s} + \int_{4m_\Lambda^2}^\infty \frac{\rho_h(s') ds'}{s' - s} - \text{subtractions} \quad (20)$$

where λ_H is defined in (2) and $g_{H\Lambda}$ is the constant for the interaction of the dihyperon H with $\Lambda\Lambda$. On the other hand, for energies which are not very large, when only the elastic channel of scattering of Λ by Λ is possible (we do not take into account the weak interactions), unitarity gives

$$f_h(s) = (1 + 2ikT_{\Lambda\Lambda}(k))f_h^*(s) \quad (21)$$

where $T_{\Lambda\Lambda} = -1/(\kappa + ik)$ is the amplitude for elastic resonance $\Lambda\Lambda$ scattering, $\kappa = \sqrt{m_\Lambda \varepsilon}$ ($\varepsilon = 2m_\Lambda - m_H$), $s = 4(m^2 + k^2)$, and $k = |\mathbf{k}|$ is the momentum of the Λ hyperon in the c.m.s. Comparing the pole behaviors of Eqs. (20) and (21), we obtain the expression for the pole of $f_h(s)$:

$$f_h(s) = \frac{\lambda_H g_{H\Lambda}}{8\kappa(\kappa + ik)} (1 + O(\kappa + ik)) . \quad (22)$$

Substituting (22) into (19), we find the required expression for the contribution of the state $|\Lambda\Lambda\rangle$ to the spectral density:

$$\rho_{\Lambda\Lambda}(s) = \frac{1}{2\pi} \frac{\lambda_H^2 g_{H\Lambda}^2}{16\kappa^2} \frac{1}{(s - 4m_\Lambda^2 + 4\kappa^2)} \frac{1}{8\pi} \left(\frac{s - 4m_\Lambda^2}{s} \right) . \quad (23)$$

The interaction constant $g_{H\Lambda}$ can be found [16] by comparing the pole contribution of the virtual dihyperon H to the quantum field-theoretical expression for the amplitude of elastic $\Lambda\Lambda$ scattering with the pole behavior of the quantum-mechanical expression for the amplitude of low-energy elastic resonance $\Lambda\Lambda$ scattering:

$$g_{H\Lambda}^2 = 32\pi 4m_\Lambda \kappa . \quad (24)$$

It can be seen from Eq. (23) that $\rho_{\Lambda\Lambda}(s)$ has a resonance peak near the $\Lambda\Lambda$ threshold with a maximum at the point $s_{max} = 4m_\Lambda^2 + 4\kappa^2$ and a width determined by the value of κ . Thus, for small κ (when the width of the resonance peak does not exceed the error in the method of QCD sum rules, i.e. 15% of m_H) this peak and the peak from the dihyperon H itself fuse in the spectral density ρ into a single effective peak, the position of whose maximum we determine by fitting the sum rule. However, since

the width of this peak for small κ does not exceed the error of the method in the determination of m_H , by calculating the position of its maximum we are calculating, within the accuracy of the method, the value of m_H itself. However, the constant λ in the spectral density (9) will then no longer be equal to the projection λ_H of the current h onto the dihyperon state [see (2)]. This will be some effective λ , whose relation of λ_H we shall now estimate.

For this, we estimate the contribution of the resonance peak from the $\Lambda\Lambda$ state to the sum rule in the leading order in κ . Using Eqs. (23) and (24), we find

$$\int_{4m_\Lambda^2}^\infty e^{-s/y} \rho_{\Lambda\Lambda}(s) ds = e^{-m_H^2/y} \lambda_H^2 \frac{1}{\pi} \frac{\sqrt{m_\Lambda E_0}}{\kappa}. \quad (25)$$

Here $s_0 = (2m_\Lambda + E_0)^2$ must be taken such that the range of integration spans the resonance peak, i.e. E_0 is approximately equal to the width of the peak. Thus, the presence of the resonance peak from the $\Lambda\Lambda$ state leads to a difference between the effective constant λ in Eq. (9) and the constant λ_H for coupling of the current h to the dihyperon H ;

$$\lambda^2 = \lambda_H^2 (1 + \pi^{-1} \sqrt{m_\Lambda E_0} / \kappa). \quad (26)$$

When κ is fairly large (i.e. when the width of the $\Lambda\Lambda$ resonance peak is larger than the uncertainty in the method of QCD sum rules in the determination of the masses), the peak becomes quite weak and can be included in the effective continuum.

Finally, we formulate our main conclusion. The analysis of our sum rules favors the existence of a dihyperon H , in other words, it agrees with Jaffe's prediction in the framework of the MIT quark-bag model that there exists a dihyperon H with the quantum numbers of two Λ hyperons. However, the accuracy of the method of QCD sum rules, which in our case is $\sim 20\%$, does not make it possible to determine whether the mass of the dihyperon H lies above or below the $\Lambda\Lambda$ threshold (i.e. whether H is stable).

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